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Screening of heavy quark free energies at finite temperature and non-zero baryon chemical potential

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Abstract. We analyze the dependence of heavy quark free energies on the baryon chemical potential μ_b in 2-flavor QCD by performing a 6th order Taylor expansion in the chemical potential which circumvents the sign problem. The bare quark mass at $\hat{m}/T = 0.4$ corresponds to a pion mass of about 770 MeV and is thus not in the range of physical quark masses but the quark mass dependence is known to be small above T_c. At $N_\tau = 4$ the lattices are coarse, however, we are using improved (p4 staggered) fermions. The Taylor expansion coefficients of color singlet and color averaged free energies are calculated and from this the expansion coefficients for the corresponding screening masses are determined. We find that for small μ_b the free energies of a static quark–antiquark pair decrease in a medium with a net excess of quarks and that screening is well described by a screening mass which increases with increasing μ_b. The μ_b-dependent corrections to the screening masses are well described by perturbation theory for $T \gtrsim 2T_c$. In particular, we find for all temperatures above T_c that the expansion coefficients for singlet and color averaged screening masses differ by a factor 2.

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1 Introduction

Numerical studies of QCD provided quite detailed information about the properties of matter at high temperature and vanishing net baryon density [1]. In particular, the screening of static quark–antiquark sources at large distances and their renormalization has been analyzed in quite some detail [2–4]. Compared to this our knowledge on the dependence of the equation of state and screening at non-zero baryon number density, or equivalently, non-zero baryon chemical potential (μ_b) is rather limited. The μ_b-dependence of the QCD partition function [5] and the HTL-resummation of the pressure [6] have been evaluated only recently. Although in leading order of high temperature perturbation theory the dependence of the Debye screening mass on μ_b is well-known [7] neither the temperature range for the validity of this perturbative result nor the generic features of screening of heavy quark free energies at non-zero μ_b have been analyzed so far with non-perturbative methods in the vicinity of the QCD transition[1].

Recently studies of the equation of state have successfully been extended to non-vanishing baryon chemical potential using Taylor expansions [9] around $\mu_b = 0$ as well as reweighting techniques [10] and imaginary chemical potentials [11]. We will use here the former approach to analyze the screening of static quark–antiquark sources at non-zero μ_b, i.e. in a medium with a non-vanishing net quark density. We evaluate the Taylor expansion coefficients for correlation functions of heavy quark–antiquark pairs and deduce from this expansion coefficients for the screening mass.

For this work we analyzed gauge field configurations using a p4-improved staggered fermion action with $N_f = 2$ degenerate quark flavors. We used the same data sample that recently has been generated by the Bielefeld–Swansea collaboration for the analysis of the equation of state [9]. This sample consists of 1000 up to 4000 gauge field configurations available for several bare gauge couplings below and above the transition temperature. The lattice size is $16^3 \times 4$ and the bare quark mass, $\hat{m}/T = 0.4$, corresponds to a pion mass of about 770 MeV. In addition we generated 1000 configurations at $T = 3T_c$ and 1600 at $T = 4T_c$ to check for the approach to the high temperature perturbative regime. In addition to gauge invariant color averaged free energies we also have analyzed color singlet free energies. To do so all gauge configurations have been transformed to Coulomb gauge before evaluating the Polyakov loop correlation functions.

[1] A first attempt to calculate heavy quark free energies at non-zero quark chemical potential has been discussed in [8].

This paper is organized as follows. In the next section we discuss the general setup for calculating Taylor expansions for heavy quark free energies. Our numerical results for singlet and color averaged free energies are presented in Sect. 3. In Sect. 4 we discuss the determination of μ_b-dependent corrections to screening masses from the free energies. Our conclusions are given in Sect. 5. In an appendix we give detailed expressions for the Taylor expansion coefficients of purely gluonic observables.

2 Taylor expansion of heavy quark free energies

A heavy (static) quark Q at site $\boldsymbol{x}$ is represented by the Polyakov loop,

$$L(\boldsymbol{x}) = \prod_{x_4=1}^{N_\tau} U_4(\boldsymbol{x}, x_4) \;, \tag{1}$$

which is an SU(3) matrix. A heavy antiquark $\bar{Q}$ is described by the corresponding hermitian conjugate matrix. The free energy of a $Q\bar{Q}$-pair separated by a distance r is then calculated from the expectation value of the correlation function of $L(\mathbf{0})$ and $L^\dagger(\boldsymbol{r})$ where $\boldsymbol{r}$ points to a site with distance r to $\mathbf{0}$. The dependence on the baryon chemical potential μ_b or quark chemical potential $\mu \equiv \mu_b/3$ is established solely through the fermion determinant, $\det M(\{U_\rho(x)\}, \mu, \hat{m})$, with $\hat{m}$ denoting the bare quark mass. In order to avoid the sign problem, which arises from $\mathrm{Re}\,(\det M)$ not being positive definite for $\mu \neq 0$, Taylor expansions in the quark chemical potential are used. This allows us to perform our simulations at zero chemical potential thereby restricting us to small chemical potentials.

A purely gluonic observable $\mathcal{O}$ like the Polyakov loop $L(\boldsymbol{x})$ or a corresponding correlation function does not explicitly depend on the quark chemical potential; it is calculated in terms of link variables $U_\rho(x)$ of the gauge field configuration which do not explicitly depend on μ. Any μ-dependence of the expectation value $\langle\mathcal{O}\rangle_\mu$ thus arises from the μ-dependence of the Boltzmann weights in the QCD partition function, i.e. the μ-dependence of the fermion determinant. Therefore we can apply the same method that was used for the power series expansion of the equation of state; expanding the fermion determinant in powers of μ leads to a power series of our purely gluonic observable,

$$\langle\mathcal{O}\rangle_\mu = \langle\mathcal{O}\rangle \cdot (1 + o_1\mu + o_2\mu^2 + \cdots), \tag{2}$$

where $\langle\mathcal{O}\rangle$ denotes the expectation value of $\mathcal{O}$ evaluated for vanishing chemical potential. We consider observables like the color averaged and singlet $Q\bar{Q}$-correlation functions,

$$\mathcal{O}^{\mathrm{av}}(r) = \frac{1}{\mathcal{N}}\frac{1}{N_c^2}\sum_{\boldsymbol{x},\boldsymbol{y}} \mathrm{Tr}L(\boldsymbol{x})\,\mathrm{Tr}L^\dagger(\boldsymbol{y}),$$
$$\mathcal{O}^{1}(r) = \frac{1}{\mathcal{N}}\frac{1}{N_c}\sum_{\boldsymbol{x},\boldsymbol{y}} \mathrm{Tr}L(\boldsymbol{x})L^\dagger(\boldsymbol{y}), \tag{3}$$

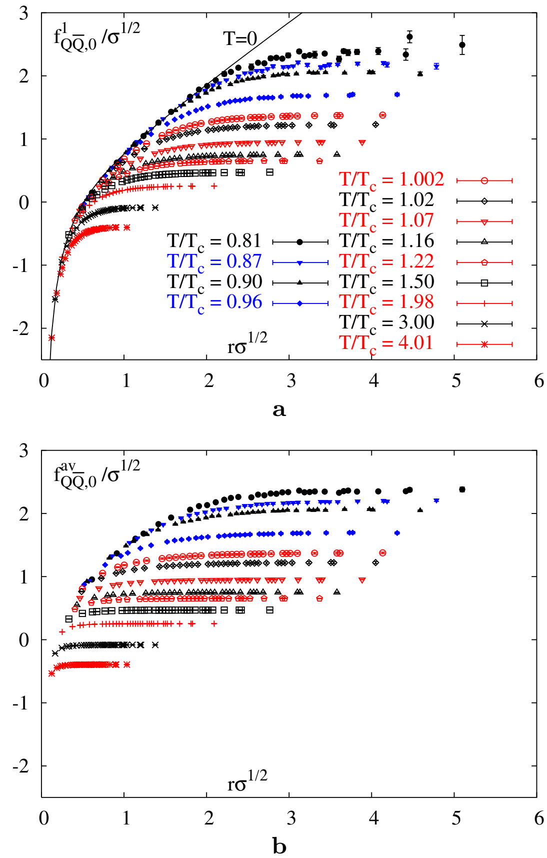

Fig. 1. The 0th order coefficients $f^1_{Q\bar{Q},0}$ and $f^{\mathrm{av}}_{Q\bar{Q},0}$ for the singlet and color averaged free energy in the vicinity of T_c. $f^1_{Q\bar{Q},0}$ is matched to the $T = 0$ heavy quark potential at small distances **a**

where the sum refers to all sites $\boldsymbol{x}, \boldsymbol{y}$ with $\|\boldsymbol{x} - \boldsymbol{y}\| = r$ and $\mathcal{N}$ is the number of these $\boldsymbol{x}, \boldsymbol{y}$-pairs. As $\mathcal{O}^{\mathrm{av},1}$ and the corresponding expectation values are strictly real for every single gauge field configuration the odd orders in the expansion vanish as was argued in [12]. For observables like the Polyakov loop itself or static quark-quark correlations like $\mathrm{Tr}L(\mathbf{0})\mathrm{Tr}L(\boldsymbol{r})$ we also have to take into account the odd orders which are in general non-vanishing. In the appendix we give explicit formulas for calculating the expansion coefficients of an arbitrary gluonic observable up to 6th order in μ/T. We have used this to evaluate the first three, non-vanishing expansion coefficients of the purely real observables considered here.

We extract the color averaged free energy of a static quark–antiquark pair from the Polyakov loop correlation function

$$F^{\mathrm{av}}_{Q\bar{Q}}(r, T, \mu) = -T\ \ln\left\langle\mathcal{O}^{\mathrm{av}}(r)\right\rangle_\mu \tag{4}$$

and the color singlet free energies from

$$F^{1}_{Q\bar{Q}}(r, T, \mu) = -T\ \ln\left\langle\mathcal{O}^{1}(r)\right\rangle_\mu. \tag{5}$$

We renormalize the Polyakov loop as described in [13] such that at short distances and vanishing chemical potential

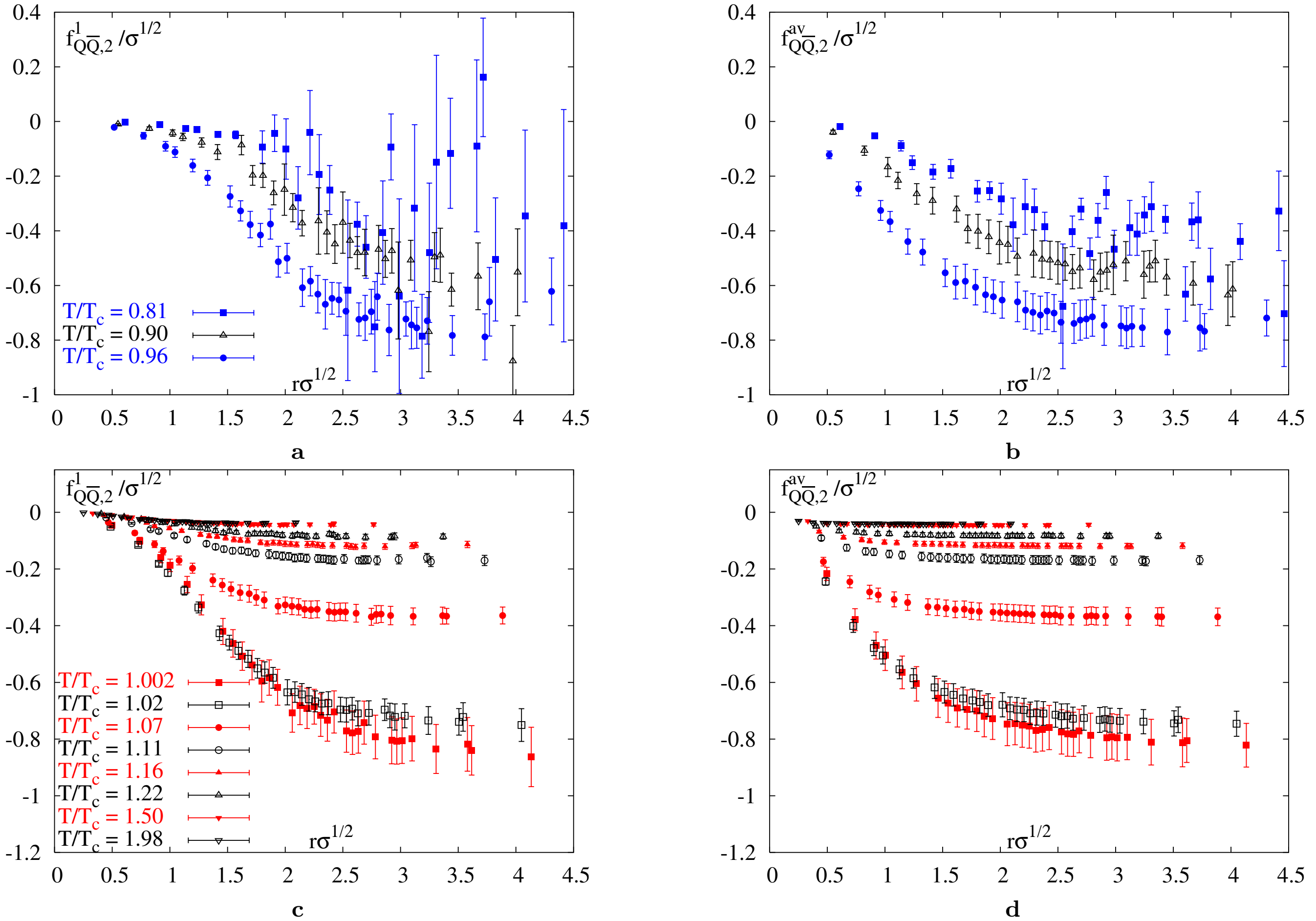

Fig. 2. The 2nd order coefficients of the singlet **a,c** and color averaged **b,d** free energies for some selected temperatures below **a,b** and above **c,d** T_c

the singlet free energy $F^1_{Q\bar{Q}}(r,T,0)$ matches the zero temperature heavy quark potential. This also fixes the renormalization of the Polyakov loop and all its correlation functions. In particular, this renormalizes also the color averaged free energies. As all our calculations have been performed on lattices with temporal extent $N_\tau = 4$ the smallest available distance at which this matching could be performed is $r_0 = 1/4T$.

In order to determine the expansion coefficients of the color averaged (av) and singlet (1) free energies,

$$\begin{aligned} &F^x_{Q\bar{Q}}(r,T,\mu) \\ &= f^x_{Q\bar{Q},0}(r,T) + f^x_{Q\bar{Q},2}(r,T)\left(\frac{\mu}{T}\right)^2 + f^x_{Q\bar{Q},4}(r,T)\left(\frac{\mu}{T}\right)^4 \\ &+ f^x_{Q\bar{Q},6}(r,T)\left(\frac{\mu}{T}\right)^6 + \mathcal{O}\left(\left(\frac{\mu}{T}\right)^8\right), \end{aligned} \quad (6)$$

with $x = \text{av}$ and 1, we apply (A.18) to the corresponding Polyakov loop correlation functions. We again note that these are strictly real on every gauge field configuration and thus have an expansion in even powers of μ/T. Explicit formulas used for the calculation of the expansion coefficients $f^{\text{av}}_{Q\bar{Q},n}(r,T)$ and $f^1_{Q\bar{Q},n}(r,T)$ are given in the appendix.

3 Numerical results on $Q\bar{Q}$ free energies

In Figs. 1–4 we show the leading and higher order expansion coefficients up to 6th order in μ/T expressed in units of the string tension[2]. We do not include data for all temperature values analyzed by us because for $T \gg T_c$ they have very small absolute values and for $T < T_c$ they suffer from large statistical errors and are still consistent with zero.

The leading order results, $f^{\text{av}}_{Q\bar{Q},0}(r,T)$ and $f^1_{Q\bar{Q},0}(r,T)$ are consistent with previous analyses of static quark–antiquark free energies performed in 2-flavor QCD at $\mu = 0$ on the same data set [3]. For the 2nd order expansion coefficients we display separately results below (Fig. 2a,b) and above (Fig. 2c,d) the $\mu = 0$ transition temperature, T_c. As can be seen the 2nd order expansion coefficients are always negative and increase in magnitude in the vicinity of T_c.

The corresponding results for the 4th and 6th order expansion coefficients are shown in Figs. 3 and 4, respec-

[2] We use $\beta_c = 3.649$ [9,12] as the bare pseudo-critical coupling. Results on the string tension [14], $a\sqrt{\sigma}$, are then used to set the temperature scale in units of T.

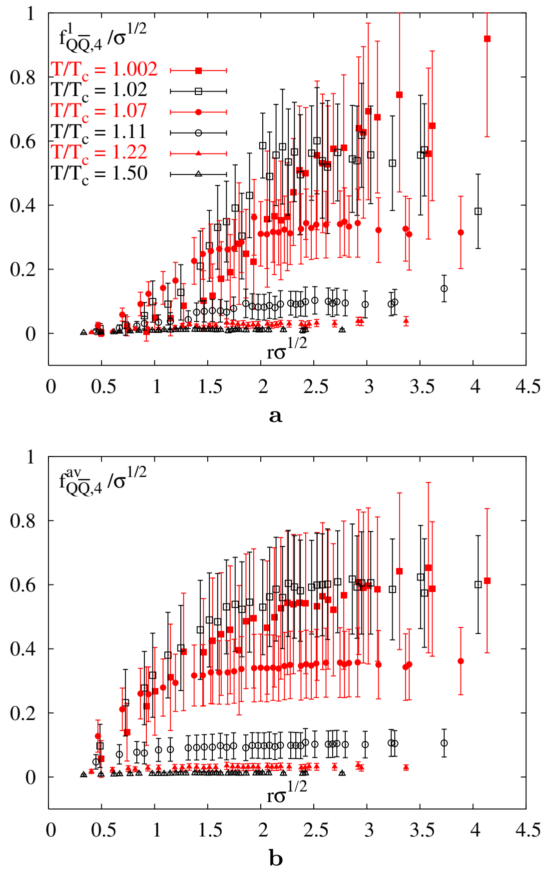

Fig. 3. The 4th order coefficients of the singlet **a** and color averaged **b** free energies for temperatures above T_c

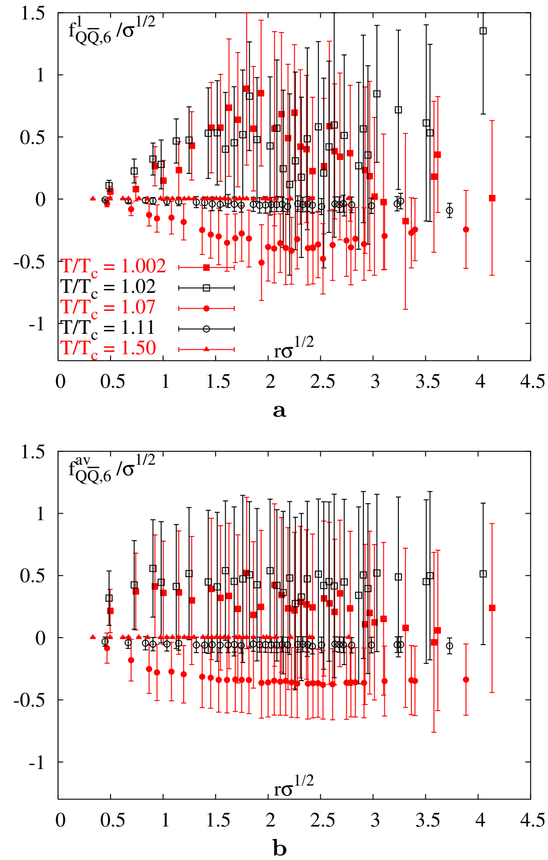

Fig. 4. The 6th order coefficients of the singlet **a** and color averaged **b** free energies for temperatures above T_c

tively. Here we only show results above T_c; below T_c the expansion coefficients are consistent with being zero within errors even at rather short distances and errors grow large for $rT \geq 1$. We note that all expansion coefficients shown in Figs. 2 to 4 vanish at small distances. This shows that a quark–antiquark pair is not affected by the surrounding medium if its size becomes small. This observation also justifies our procedure to renormalize the Polyakov loop by matching the $\mu = 0$ singlet free energy to the $T = 0$ heavy quark potential. The renormalization constant is independent of μ.

Also close to T_c, where the μ-dependence of the free energies is strongest, the absolute values of the 4th and 6th order expansion coefficients are of the same order as or smaller than the 2nd order expansion coefficient. Therefore the 4th and 6th order contributions rapidly become negligible for $\mu/T < 1$.

Being the results of quite a few terms with opposite signs the errors are large for the higher order expansion coefficients especially at 6th order. Nevertheless they show that at high temperature the 2nd and 4th order expansion coefficients are opposite in sign, $f^{\mathrm{av},1}_{Q\bar{Q},2}(r,T) < 0$ and $f^{\mathrm{av},1}_{Q\bar{Q},4}(r,T) > 0$. This is consistent with the expectation that at high temperature the asymptotic large distance value of the heavy quark free energy is proportional to the value of the Debye mass [15]. In this limit one obtains alternating signs of the expansion coefficients of the heavy quark free energies when one expands the perturbative Debye mass [7],

$$\frac{m_\mathrm{D}(T,\mu)}{g(T)T} = \frac{m_{\mathrm{D},0}(T)}{g(T)T}\sqrt{1 + \frac{3N_f}{(2N_c+N_f)\pi^2}\left(\frac{\mu}{T}\right)^2}\,, \quad (7)$$

with $m_{\mathrm{D},0}(T) = g(T)T\sqrt{\frac{N_c}{3}+\frac{N_f}{6}}$ denoting the Debye mass for vanishing baryon chemical potential. Although the statistical significance of our results for $f^{\mathrm{av},1}_{Q\bar{Q},6}(r,T)$ rapidly drops with increasing temperature this pattern of alternating signs seems to be valid also at 6th order at least for temperatures $T \gtrsim 1.05T_c$.

Except for temperatures close to the transition temperature the asymptotic behavior of the free energies is reached at distances $rT \gtrsim 1.5$. We determined their large distance value by taking the weighted average of the values at the five largest distances. The results are shown in Fig. 5. We note that $|f^{\mathrm{av},1}_{Q\bar{Q},2}(\infty,T)|$ have a pronounced peak at T_c. This also holds for $|f^{\mathrm{av},1}_{Q\bar{Q},2}(r,T)|$ evaluated at any fixed distance r. In fact, $f^{\mathrm{av},1}_{Q\bar{Q},2}(r,T)$ is proportional to the 2nd derivative of a partition function including a pair of static sources, $Q\bar{Q}$. It thus shows the characteristic properties of a susceptibility in the vicinity of a (phase) transition point.

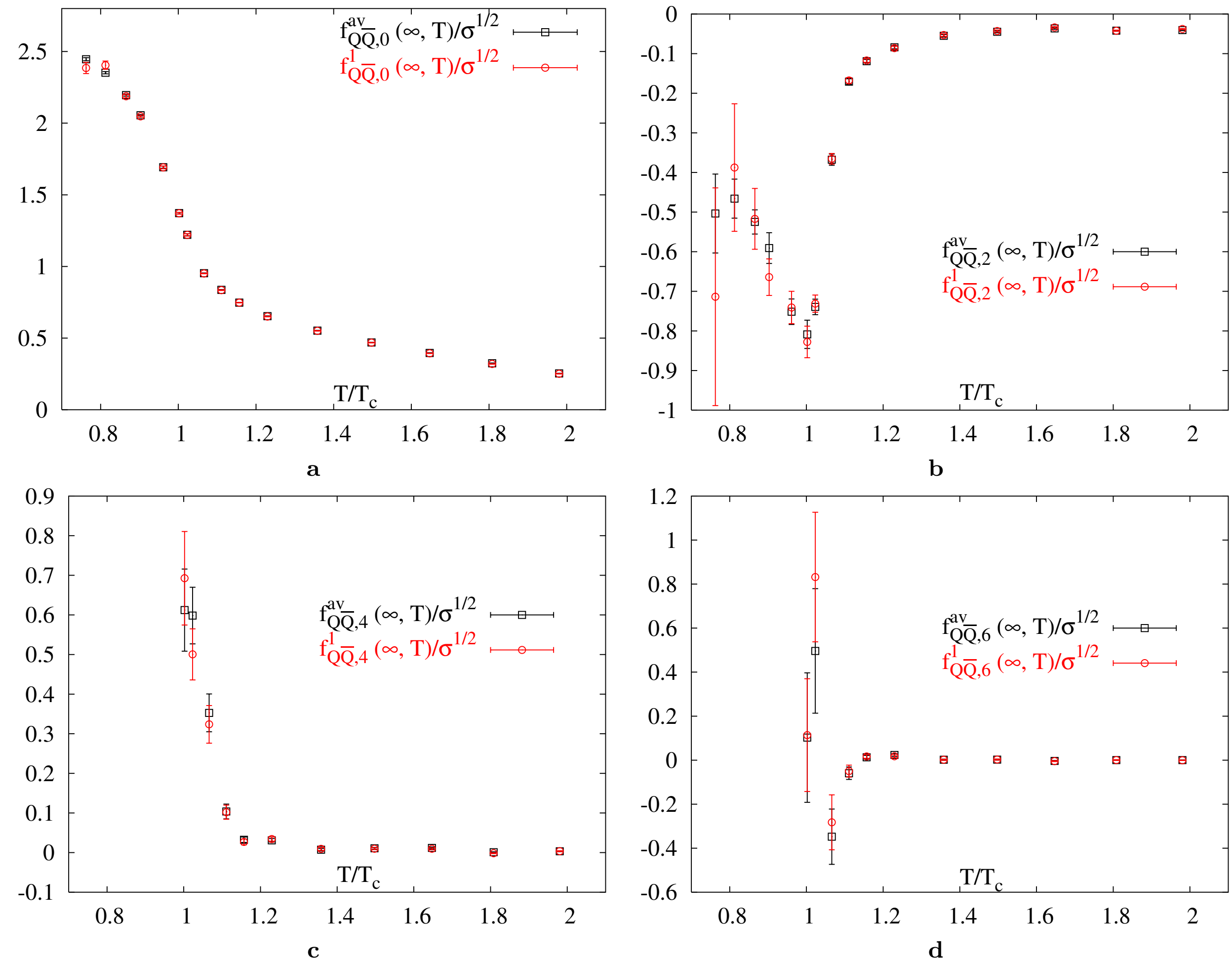

Fig. 5. The coefficients for the singlet and color averaged free energies at infinite distance rT versus temperature. They have been obtained from a weighted average of $f^{\mathrm{av}}_{Q\bar{Q},n}(r,T)$ and $f^{1}_{Q\bar{Q},n}(r,T)$ at the five largest distances

Figure 5 also shows that at large distances, within the statistical errors of our analysis, the expansion coefficients for the color averaged and singlet free energies approach identical values,

$$f^{\mathrm{av}}_{Q\bar{Q},n}(\infty,T) = f^{1}_{Q\bar{Q},n}(\infty,T), \tag{8}$$

where

$$f^{x}_{Q\bar{Q},n}(\infty,T) = \lim_{r\to\infty} f^{x}_{Q\bar{Q},n}(r,T)\ , \quad \text{with } x = \mathrm{av},\ 1. \tag{9}$$

This has been noted before at $\mu = 0$ and suggests that at large distances, e.g. for $rT \gtrsim 1.5$, the quark–antiquark sources are screened independently from each other; their relative color orientation thus becomes irrelevant.

Including all terms up to 6th order we calculated the singlet and color averaged free energies in the range from $\mu/T = 0.0$ up to 0.8. Results for the color singlet free energies evaluated at a few values of temperature are shown in Fig. 6. Similar results hold for the color averaged free energies. The free energies decrease relative to their values at $\mu/T = 0$ for all temperatures above and below T_c. At small distances the curves always agree within errors. With increasing distance a gap opens up which reflects the decrease in free energy at non-zero μ. As indicated by the asymptotic values $f^{\mathrm{av},1}_{Q\bar{Q},2}(\infty,T)$, which give the dominant μ-dependent contribution at large distances, the medium effects are largest close to the transition temperature and become smaller with increasing temperature.

4 Screening masses

For temperatures above T_c and large distances r the heavy quark free energies are expected to be screened,

$$\begin{aligned}\Delta F^{\mathrm{av},1}_{Q\bar{Q}}(r,T,\mu) &= F^{\mathrm{av},1}_{Q\bar{Q}}(\infty,T,\mu) - F^{\mathrm{av},1}_{Q\bar{Q}}(r,T,\mu)\\ &\sim \frac{1}{r^n}\mathrm{e}^{-m^{\mathrm{av},1}(T,\mu)r},\end{aligned} \tag{10}$$

with $n = 1, 2$ for the singlet and color averaged free energies respectively. In the infinite distance limit we thus can

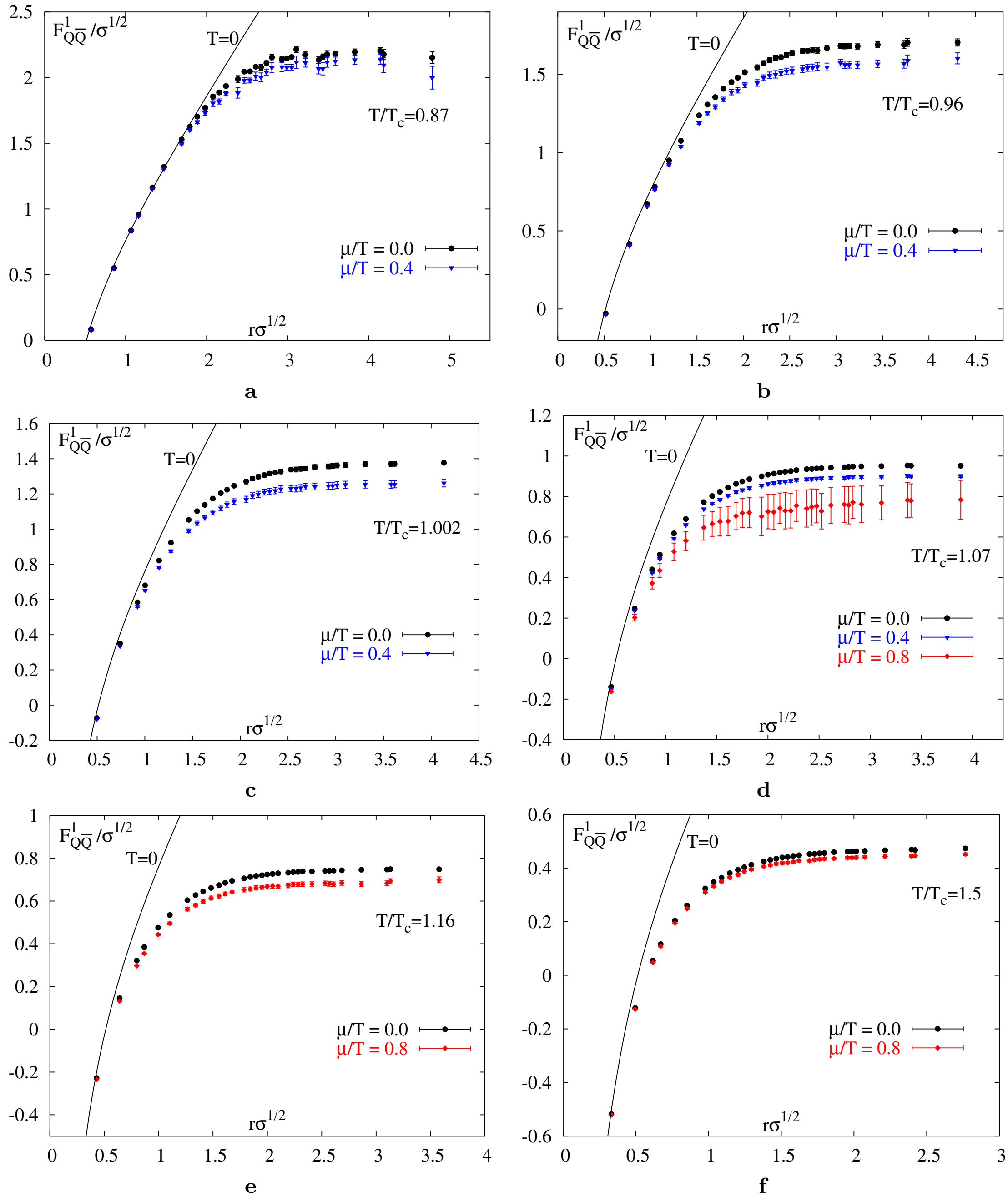

Fig. 6. The singlet free energies $F^1_{Q\bar{Q}}$ as function of distance for finite chemical potential and for various temperatures

extract the screening masses,

$$m^{\mathrm{av},1}(T,\mu) = -\lim_{r\to\infty} \frac{1}{r} \ln\left(\Delta F^{\mathrm{av},1}_{Q\bar{Q}}(r,T,\mu)\right) . \quad (11)$$

We use this as our starting point to derive a Taylor expansion for the screening masses. Expanding the logarithm in (11) in powers of μ/T it is obvious that also the screening masses are even functions in μ/T,

$$\begin{aligned} &m^{\mathrm{av},1}(T,\mu) \\ &= m_0^{\mathrm{av},1}(T) + m_2^{\mathrm{av},1}(T)\left(\frac{\mu}{T}\right)^2 + m_4^{\mathrm{av},1}(T)\left(\frac{\mu}{T}\right)^4 \\ &\quad + m_6^{\mathrm{av},1}(T)\left(\frac{\mu}{T}\right)^6 + \mathcal{O}\left(\left(\frac{\mu}{T}\right)^8\right) . \end{aligned} \quad (12)$$

To analyze the approach of the various expansion coefficients to the large distance limits we introduce effective masses, $m^x_{\mathrm{eff},n}(r,T)$, with $x = \mathrm{av}, 1$,

$$m^x_{\mathrm{eff},2}(r,T) = -\frac{1}{r}\frac{\Delta f^x_{Q\bar{Q},2}(r,T)}{\Delta f^x_{Q\bar{Q},0}(r,T)}, \quad (13a)$$

$$m^x_{\mathrm{eff},4}(r,T)$$

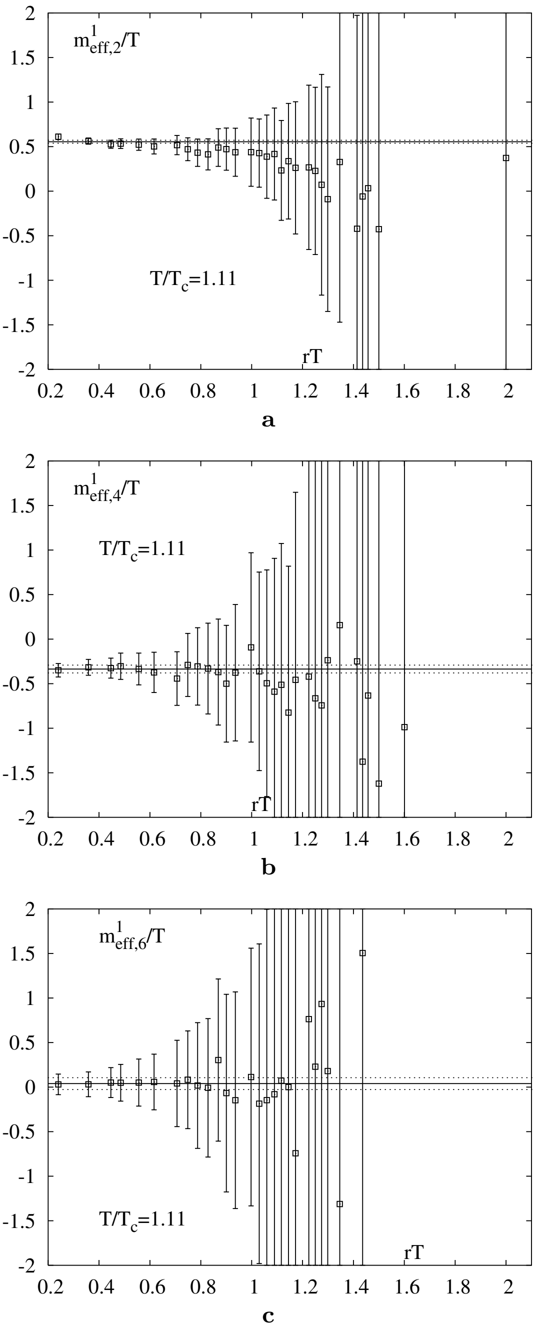

Fig. 7. Example for the distance dependent effective masses converging at large distance to the expansion coefficients for the screening mass which are represented by the horizontal lines

$$= -\frac{1}{r}\left[\frac{\Delta f^x_{Q\bar{Q},4}(r,T)}{\Delta f^x_{Q\bar{Q},0}(r,T)} - \frac{1}{2}\left(\frac{\Delta f^x_{Q\bar{Q},2}(r,T)}{\Delta f^x_{Q\bar{Q},0}(r,T)}\right)^2\right] , \quad (13b)$$

$m^x_{\mathrm{eff},6}(r,T)$

$$= -\frac{1}{r}\left[\frac{\Delta f^x_{Q\bar{Q},6}(r,T)}{\Delta f^x_{Q\bar{Q},0}(r,T)} - \frac{\Delta f^x_{Q\bar{Q},4}(r,T)\Delta f^x_{Q\bar{Q},2}(r,T)}{\Delta f^x_{Q\bar{Q},0}(r,T)^2}\right.$$

$$\left. + \frac{1}{3}\left(\frac{\Delta f^x_{Q\bar{Q},2}(r,T)}{\Delta f^x_{Q\bar{Q},0}(r,T)}\right)^3\right] . \quad (13c)$$

In the limit of large distances these relations define the expansion coefficients of the color averaged and singlet screening masses,

$$m_n^{\mathrm{av},1}(T) = \lim_{r\to\infty} m^{\mathrm{av},1}_{\mathrm{eff},n}(r,T) . \quad (14)$$

As will become obvious in the following the effective masses defined above show only little r-dependence. They are thus suitable for a determination of the μ-dependent corrections to the screening masses. This is not the case for the leading order, μ-independent, contribution. In order to determine $m_0^1(T)$ we use an ansatz for the large distance behavior of the singlet free energy motivated by leading order high temperature perturbation theory,

$$f^1_{Q\bar{Q},0}(r,T) = f^1_{Q\bar{Q},0}(\infty,T) - \frac{4}{3}\frac{\alpha_0(T)}{r}\mathrm{e}^{-m_0^1(T)r} . \quad (15)$$

We fit our data to this equation using $\alpha_0(T)$ and $m_0^1(T)$ as fit parameters where $f^1_{Q\bar{Q},0}(\infty,T)$ is determined as described in the previous section. We choose the same fitting procedure as in [3] namely averaging results received from five fit windows with left borders between $rT = 0.8$ and $rT = 1.0$ and right border at $rT = 1.73$. While the above ansatz is known to describe rather well the large distance behavior of the color singlet free energy, it also is known that the sub-leading power-like corrections are much more difficult to control in the case of the color averaged free energy. For this reason we will analyze here only the leading order contribution to the singlet screening mass.

Results for effective masses in the singlet channel are shown in Fig. 7 as function of rT for one value of the temperature. As can be seen the asymptotic value is indeed reached quickly before the errors grow large at distances $rT \gtrsim 1$. The expansion coefficients $m_2^{\mathrm{av},1}(T)$, $m_4^{\mathrm{av},1}(T)$ and $m_6^{\mathrm{av},1}(T)$ are thus well determined from the plateau values of these ratios. Similar results hold in the color averaged channel. We found the left border of the plateau to lie between $rT = 0.48$ close to T_c and $rT = 0.23$ for $T > 1.15T_\mathrm{c}$. Results for the various expansion coefficients are shown in Fig. 8. This figure shows that at high temperatures the μ-dependent corrections to the screening mass of the color averaged free energies $m^{\mathrm{av}}(T,\mu)$ are twice as large as those of the (Debye) screening mass in the singlet channel, $m^1(T,\mu)$. This is expected from perturbation theory, which suggests that the leading order contribution to the color singlet free energy is given by one gluon exchange while the color averaged free energy is dominated by two gluon exchange. Using resummed gluon propagators then leads to screening masses that differ by a factor of 2,

$$m_n^1(T) = \frac{1}{2}m_n^{\mathrm{av}}(T), \quad n = 2,4,6 , \quad (16)$$

Our results suggest that this relation holds already close to T_c (Fig. 8). We thus have no evidence for large contributions from the magnetic sector, which is expected to

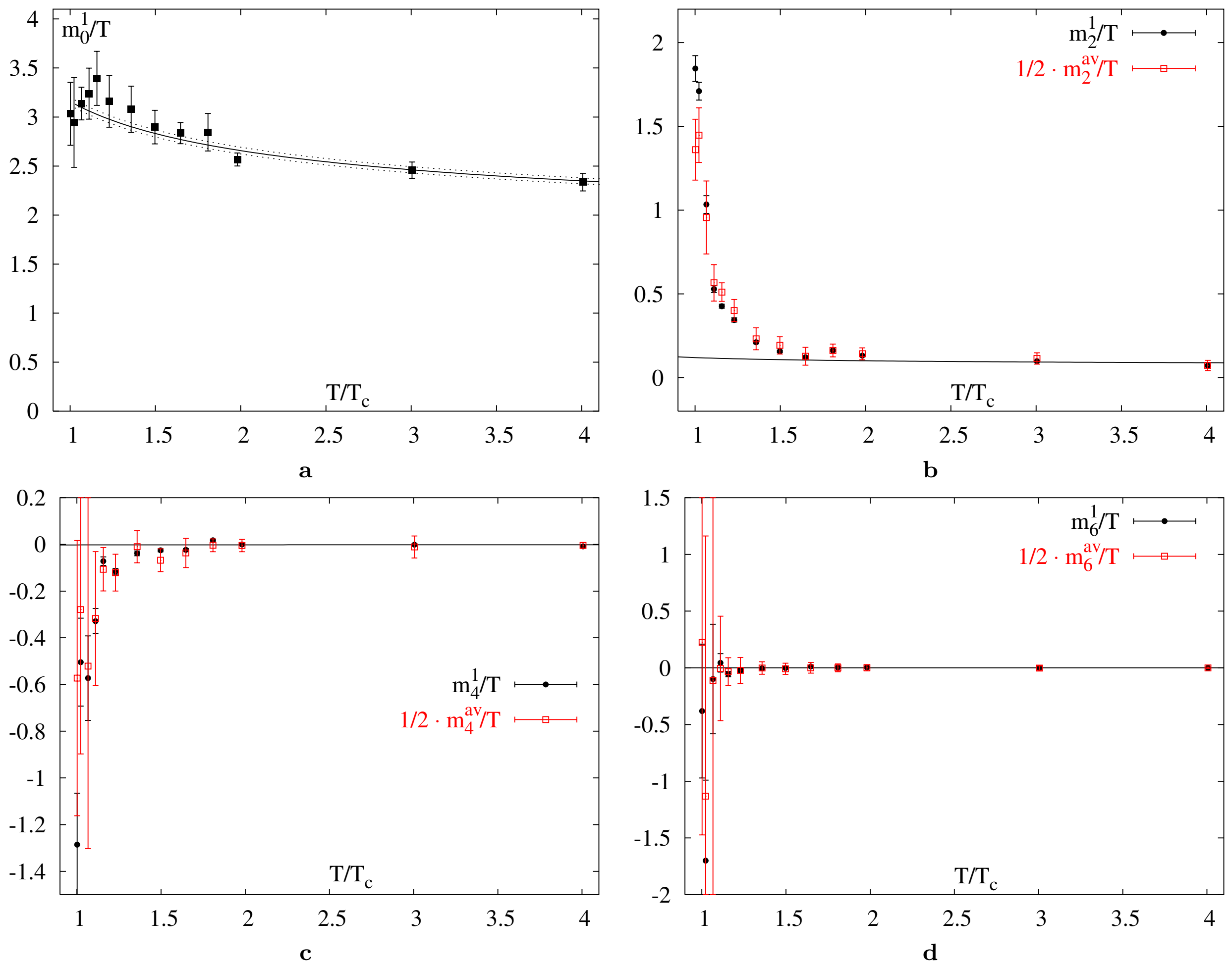

Fig. 8. Expansion coefficients of screening masses in the color singlet (m_n^1) and color averaged ($m_n^{\rm av}$) channels. Except for $n = 0$ $m_n^x(T)$ is determined from the $r \to \infty$ limit of (13). The lines are the first order perturbative predictions according to (20). The dotted lines in **a** show the 1σ-range of the χ^2-fit with parameter A

dominate the screening in the color averaged channel at asymptotically large temperatures [16] and which would violate the simple relation given in (16).

In order to compare the expansion coefficients with perturbation theory we need to specify the running coupling $g(T)$. Following [3] we use the next-to-leading order perturbative result for the running of the coupling with temperature but allow for a free overall scale factor. We thus fit our data on the T-dependence of the leading order ($\mu = 0$) screening mass by the ansatz

$$m_0^1(T) = A \cdot \frac{2}{\sqrt{3}} g(T) T \;, \tag{17}$$

with the 2nd order perturbative running coupling,

$$g(T)^2 = \left[\frac{29}{24\pi^2}\ln\left(\frac{\widetilde{\mu}T}{\Lambda_{\overline{MS}}}\right) + \frac{115}{232\pi^2}\ln\left(\ln\left(\frac{\widetilde{\mu}T}{\Lambda_{\overline{MS}}}\right)\right)\right]^{-1}, \tag{18}$$

where we use $T_c/\Lambda_{\overline{MS}} = 0.77(21)$ and the scale $\widetilde{\mu} = 2\pi$ as in [3]. Fitting our data to (17) with fit parameter A yields

$$A = 1.397(18), \tag{19a}$$

which is almost identical to the result in [3] where the data for $T = 3T_c$ were still missing. Our fit result is included in Fig. 8. We also compare the temperature dependence of $m_2^1(T)$, $m_4^1(T)$ and $m_6^1(T)$ with corresponding expansion coefficients of the perturbative Debye mass which result from an expansion of (7) using (17) as the 0th order. These expansion coefficients are alternating in sign,

$$m_{D,2}(T) = \frac{\sqrt{3}}{4\pi^2} \cdot A g(T), \tag{20a}$$

$$m_{D,4}(T) = -\frac{3\sqrt{3}}{64\pi^4} \cdot A g(T), \tag{20b}$$

$$m_{D,6}(T) = \frac{9\sqrt{3}}{512\pi^6} \cdot A g(T). \tag{20c}$$

At least for the 2nd order coefficient $m_2^1(T)$ we find that this yields a satisfactory description of the numerical results for $T \gtrsim 2T_c$. Equation (20) shows that subsequent

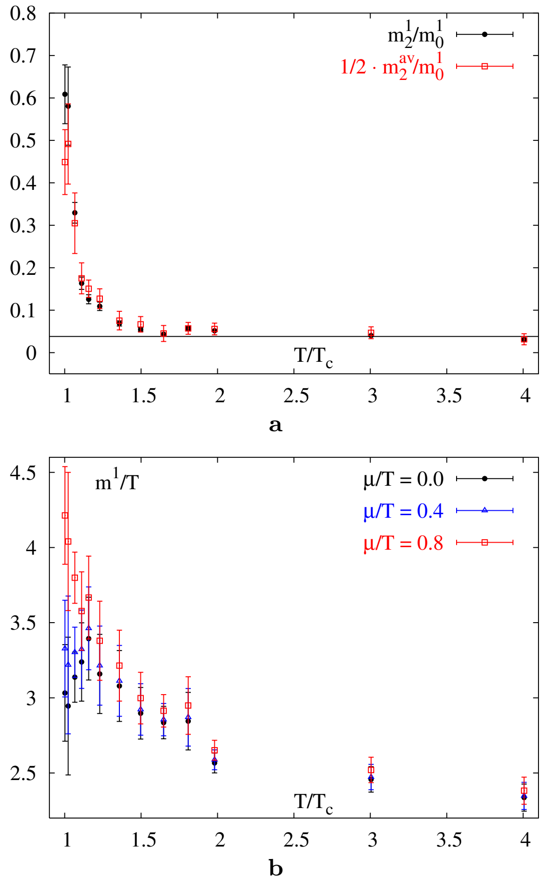

Fig. 9. The ratio $m_2^x(T)/m_0^1(T)$ (a) and the screening mass $m^1(T)$ evaluated for $\mu/T = 0.0, 0.4$ and 0.8 including the 0th and 2nd order expansion coefficients in μ (b). The line in (a) shows the leading order perturbative prediction, which is $3/8\pi^2$

terms differ by about an order of magnitude, which explains why our signal for a non-zero contribution $m_n^1(T)$ is rather poor for $n > 2$.

From (20) we find $m_{\mathrm{D},2}(T)/m_{\mathrm{D},0}(T) = 3/8\pi^2$ which is independent of A and $g(T)$ and is compared with our numerical results in Fig. 9a. We note that the perturbative value for this ratio is already reached for $T/T_\mathrm{c} \gtrsim 2$. In Fig. 9b we show the μ-dependence of the singlet screening mass for small values of μ/T. Here we included only contributions from the 0th and 2nd order expansion in the calculation of $m^1(\mu, T)/T$.

5 Conclusions

We have analyzed the response of color singlet and color averaged heavy quark free energies to a non-vanishing baryon chemical potential and have calculated the resulting dependence of screening masses on the chemical potential. Using a Taylor expansion in μ/T we get stable results for the leading, non-vanishing correction, $m_2^1(T)$, which is $\mathcal{O}((\mu/T)^2)$. We find that this correction in absolute units as well as its ratio with the leading order screening mass, $m_0^1(T)$, is large in the vicinity of the transition temperature. The ratio $m_2^1(T)/m_0^1(T)$ is in agreement with perturbation theory for $T \gtrsim 2T_\mathrm{c}$ indicating that the expansion coefficients $m_n^1(T)$ receive the same multiplicative rescaling as the leading order screening mass.

A calculation of the μ-dependent corrections to the screening mass in the color averaged channel shows that these corrections are twice as large as those in the color singlet channel for all temperatures $T > T_\mathrm{c}$. This agreement with leading order perturbation theory indeed is quite remarkable as it suggests that the leading contribution to the μ-dependent corrections of the color averaged screening mass is due to two-gluon exchange.

The higher order expansion coefficients of the screening mass vanish within statistical errors at temperatures larger than $1.2T_\mathrm{c}$. The analysis of the asymptotic behavior of the free energies themselves, however, suggests that these corrections are non-zero but small at high temperature and have alternating signs. This is consistent with the leading order perturbative result for the Debye mass subsequent expansion coefficients of which drop by more than an order of magnitude and alternate in sign as they arise from an expansion of a square root.

Our results thus suggest that at least for small values of the chemical potential and fixed temperature the screening length in a baryon rich quark gluon plasma decreases with increasing value of the chemical potential. This is consistent with the expectation that the transition to the high temperature phase shifts to lower temperatures at non-zero baryon chemical potential.

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Appendix A: Calculation of expansion coefficients

The μ-dependent expectation value of a complex quantity $\mathcal{O}$ is

$$\langle \mathcal{O} \rangle_\mu = \frac{1}{Z_\mu} \int DU \mathcal{O} \varDelta \mathrm{e}^{-S} = \frac{\int DU \mathcal{O} \varDelta \mathrm{e}^{-S}}{\int DU \varDelta \mathrm{e}^{-S}}, \quad \text{(A.1)}$$

where Z_μ is the partition function for finite μ and where $\varDelta = (\det M(\mu))^{N_f/4}$ is the determinant of the fermion matrix. In the following we denote expectation values for vanishing μ as $\langle \cdots \rangle = \langle \cdots \rangle_0$. We define

$$L_n \equiv \left. \frac{\mathrm{d}^n \ln \varDelta}{\mathrm{d}\mu^n} \right|_{\mu=0} = \frac{N_f}{4} \left. \frac{\mathrm{d}^n \ln \det M}{\mathrm{d}\mu^n} \right|_{\mu=0}. \quad \text{(A.2)}$$

The L_n can be written as traces over the inverse of the fermion matrix and its derivatives

$$L_1 = \frac{N_f}{4} \mathrm{Tr} \left. \left(M^{-1} \frac{\partial M}{\partial \mu} \right) \right|_{\mu=0}, \quad \text{(A.3)}$$

$$L_2 = \frac{N_f}{4}\mathrm{Tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2} - M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}\right)\Bigg|_{\mu=0}, \tag{A.4}$$

$$\begin{aligned}L_3 = \frac{N_f}{4}\mathrm{Tr}\Bigg(&M^{-1}\frac{\partial^3 M}{\partial\mu^3} - 3M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\\ &+ 2M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}\Bigg)\Bigg|_{\mu=0},\end{aligned} \tag{A.5}$$

$$\begin{aligned}L_4 = \frac{N_f}{4}\mathrm{Tr}\Bigg(&M^{-1}\frac{\partial^4 M}{\partial\mu^4} - 4M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\\ &- 3M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\\ &+ 12M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\\ &- 6M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}\Bigg)\Bigg|_{\mu=0},\end{aligned} \tag{A.6}$$

$$\begin{aligned}L_5 = \frac{N_f}{4}\mathrm{Tr}\Bigg(&M^{-1}\frac{\partial^5 M}{\partial\mu^5} - 5M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^4 M}{\partial\mu^4}\\ &- 10M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\\ &+ 20M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\\ &+ 30M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\\ &- 60M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\\ &+ 24M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}\\ &\times M^{-1}\frac{\partial M}{\partial\mu}\Bigg)\Bigg|_{\mu=0},\end{aligned} \tag{A.7}$$

$$\begin{aligned}L_6 = \frac{N_f}{4}\mathrm{Tr}\Bigg(&M^{-1}\frac{\partial^6 M}{\partial\mu^6} - 6M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^5 M}{\partial\mu^5}\\ &- 15M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^4 M}{\partial\mu^4}\\ &- 10M^{-1}\frac{\partial^3 M}{\partial\mu^3}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\\ &+ 30M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^4 M}{\partial\mu^4}\\ &+ 60M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\\ &+ 60M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\\ &+ 30M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\\ &- 120M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\\ &- 180M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\\ &- 90M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\\ &+ 360M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\\ &- 120M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}\\ &\times M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}\Bigg)\Bigg|_{\mu=0}.\end{aligned} \tag{A.8}$$

From

$$M^\dagger(\mu) = \gamma_5 M(-\mu)\gamma_5 \tag{A.9}$$

it follows that L_n is real for even and imaginary for odd n. Using $\Delta = \mathrm{e}^{\ln\Delta}$ we find

$$\begin{aligned}&\Delta(\mu)\\ &= \Delta(0)\left(1 + D_1\mu + D_2\mu^2 + \cdots + D_6\mu^6 + \mathcal{O}(\mu^7)\right),\end{aligned} \tag{A.10}$$

where

$$D_1 = L_1, \tag{A.11a}$$

$$D_2 = \frac{1}{2}\left(L_1^2 + L_2\right), \tag{A.11b}$$

$$D_3 = \frac{1}{6}\left(L_1^3 + 3L_1L_2 + L_3\right), \tag{A.11c}$$

$$D_4 = \frac{1}{24}\left(L_1^4 + 6L_1^2L_2 + 3L_2^2 + 4L_1L_3 + L_4\right), \tag{A.11d}$$

$$\begin{aligned}D_5 = \frac{1}{120}&\left(L_1^5 + 10L_1^3L_2 + 15L_1L_2^2 + 10L_1^2L_3\right.\\ &\left.+ 10L_2L_3 + 5L_1L_4 + L_5\right),\end{aligned} \tag{A.11e}$$

$$\begin{aligned}D_6 = \frac{1}{720}&\left(L_1^6 + 15L_1^4L_2 + 45L_1^2L_2^2 + 15L_2^3\right.\\ &+ 20L_1^3L_3 + 60L_1L_2L_3\\ &\left.+ 10L_3^2 + 15L_1^2L_4 + 15L_2L_4 + 6L_1L_5 + L_6\right).\end{aligned} \tag{A.11f}$$

We immediately see that D_n is real for even and imaginary for odd n. Because

$$Z_\mu = \left\langle 1 + D_1\mu + \cdots D_6\mu^6\right\rangle + \mathcal{O}(\mu^7) \tag{A.12}$$

is real one has $\langle D_n\rangle = 0$ for odd n. We consider the case where the observable $\mathcal{O}$ is independent of μ. The expectation value (A.1) then becomes

$$\begin{aligned}&\langle\mathcal{O}\rangle_\mu\\ &= \frac{\langle\mathcal{O}\rangle + \langle\mathcal{O}D_1\rangle\mu + \cdots + \langle\mathcal{O}D_6\rangle\mu^6}{1 + \langle D_2\rangle\mu^2 + \cdots + \langle D_6\rangle\mu^6} + \mathcal{O}(\mu^7).\end{aligned} \tag{A.13}$$

Expanding in powers of μ we get

$$\begin{aligned}\langle\mathcal{O}\rangle_\mu = \langle\mathcal{O}\rangle&\left(1 + \mathcal{O}_1\mu + (-\mathcal{D}_2 + \mathcal{O}_2)\mu^2\right.\\ &+ (-\mathcal{D}_2\mathcal{O}_1 + \mathcal{O}_3)\mu^3\\ &+ \left(\mathcal{D}_2^2 - \mathcal{D}_4 - \mathcal{D}_2\mathcal{O}_2 + \mathcal{O}_4\right)\mu^4\\ &+ \left(\mathcal{D}_2^2\mathcal{O}_1 - \mathcal{D}_4\mathcal{O}_1 - \mathcal{D}_2\mathcal{O}_3 + \mathcal{O}_5\right)\mu^5\\ &+ \left(-\mathcal{D}_2^3 + 2\mathcal{D}_2\mathcal{D}_4 - \mathcal{D}_6 + \mathcal{D}_2^2\mathcal{O}_2 - \mathcal{D}_4\mathcal{O}_2\right.\end{aligned}$$

$$- \mathcal{D}_2\mathcal{O}_4 + \mathcal{O}_6)\,\mu^6\big) + \mathcal{O}(\mu^7), \qquad (A.14)$$

where we use the notation

$$\mathcal{O}_i = \frac{\langle \mathcal{O} D_i \rangle}{\langle \mathcal{O} \rangle}, \qquad (A.15a)$$

$$\mathcal{D}_i = \langle D_i \rangle\,. \qquad (A.15b)$$

In the case that $\mathcal{O}$ is strictly real on every configuration $\mathcal{O}D_n$ is imaginary for odd n. In order to keep $\langle \mathcal{O} \rangle_\mu$ real $\langle \mathcal{O}D_n \rangle$ has to vanish for odd n and the preceding expansion simplifies to

$$\begin{aligned}\langle \mathcal{O} \rangle_\mu = \langle \mathcal{O} \rangle \big(1 &+ (-\mathcal{D}_2 + \mathcal{O}_2)\,\mu^2 \\ &+ \big(\mathcal{D}_2^2 - \mathcal{D}_4 - \mathcal{D}_2\mathcal{O}_2 + \mathcal{O}_4\big)\,\mu^4 \\ &+ \big(-\mathcal{D}_2^3 + 2\mathcal{D}_2\mathcal{D}_4 - \mathcal{D}_6 + \mathcal{D}_2^2\mathcal{O}_2 - \mathcal{D}_4\mathcal{O}_2 \\ &- \mathcal{D}_2\mathcal{O}_4 + \mathcal{O}_6)\,\mu^6\big) + \mathcal{O}(\mu^8),\end{aligned} \qquad (A.16)$$

i.e. this formula is applicable to the correlation function in (3).

Because free energies are calculated from logarithms of correlation functions we give here the expansion coefficients of the logarithm of an observable $\mathcal{O}$ which can be obtained by inserting the above expansion into the expansion of the logarithm. For a generic, not necessarily real observable the expansion is

$$\begin{aligned}&\ln \langle \mathcal{O} \rangle_\mu \\ &= \ln \langle \mathcal{O} \rangle + \mathcal{O}_1\mu + \left(-\mathcal{D}_2 - \frac{1}{2}\mathcal{O}_1^2 + \mathcal{O}_2\right)\mu^2 \\ &+ \left(\frac{1}{3}\mathcal{O}_1^3 - \mathcal{O}_1\mathcal{O}_2 + \mathcal{O}_3\right)\mu^3 \\ &+ \Big(\frac{1}{2}\mathcal{D}_2^2 - \mathcal{D}_4 - \frac{1}{4}\mathcal{O}_1^4 + \mathcal{O}_1^2\mathcal{O}_2 - \frac{1}{2}\mathcal{O}_2^2 \\ &- \mathcal{O}_1\mathcal{O}_3 + \mathcal{O}_4\Big)\,\mu^4 \\ &+ \Big(\frac{1}{5}\mathcal{O}_1^5 - \mathcal{O}_1^3\mathcal{O}_2 + \mathcal{O}_1\mathcal{O}_2^2 + \mathcal{O}_1^2\mathcal{O}_3 - \mathcal{O}_2\mathcal{O}_3 \\ &- \mathcal{O}_1\mathcal{O}_4 + \mathcal{O}_5\Big)\,\mu^5 \\ &+ \Big(-\frac{1}{3}\mathcal{D}_2^3 + \mathcal{D}_2\mathcal{D}_4 - \mathcal{D}_6 - \frac{1}{6}\mathcal{O}_1^6 + \mathcal{O}_1^4\mathcal{O}_2 \\ &- \frac{3}{2}\mathcal{O}_1^2\mathcal{O}_2^2 + \frac{1}{3}\mathcal{O}_2^3 - \mathcal{O}_1^3\mathcal{O}_3 + 2\mathcal{O}_1\mathcal{O}_2\mathcal{O}_3 - \frac{1}{2}\mathcal{O}_3^2 \\ &+ \mathcal{O}_1^2\mathcal{O}_4 - \mathcal{O}_2\mathcal{O}_4 - \mathcal{O}_1\mathcal{O}_5 + \mathcal{O}_6\Big)\,\mu^6 + \mathcal{O}(\mu^7).\end{aligned} \qquad (A.17)$$

For real observables this reduces to

$$\begin{aligned}&\ln \langle \mathcal{O} \rangle_\mu = \ln \langle O \rangle \\ &+ (-\mathcal{D}_2 + \mathcal{O}_2)\,\mu^2 + \left(\frac{1}{2}\mathcal{D}_2^2 - \mathcal{D}_4 - \frac{1}{2}\mathcal{O}_2^2 + \mathcal{O}_4\right)\mu^4 \\ &+ \left(-\frac{1}{3}\mathcal{D}_2^3 + \mathcal{D}_2\mathcal{D}_4 - \mathcal{D}_6 + \frac{1}{3}\mathcal{O}_2^3 - \mathcal{O}_2\mathcal{O}_4 + \mathcal{O}_6\right)\mu^6 \\ &+ \mathcal{O}(\mu^8).\end{aligned} \qquad (A.18)$$

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